How to Build the Perfect Igloo

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Building an igloo, or dome in general, is a task humanity has faced since antiquity. The chord lengths of geodesic domes were considered classified military information in the United States until the sixties, and some believe that the secrets of medieval cathedral dome builders form the origins of Freemasonry Even now, the construction is not an easy procedure.

The Inuit are known for their ability to build snow domes. They build layers of bricks in a spiral pattern, causing the dome to close in loxodromically (see Figure 1) Due to the multitude of different brick shapes, this method is rather difficult for the amateur to carry out.

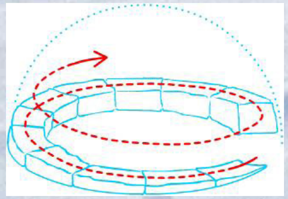

Figure 1 The Inuit method for building an igloo

Mathematical Formulation

In developing an easier process for igloo-building, we are interested the following question: *is it possible to split the spherical dome into identical elements*? The answer is yes, of course. For example, we can cut the dome into n slices using lines of longitude, forming spherical triangles with two right angles at the base. Such a form would not be very useful for our purposes though. We must impose additional requirements on our block forms, with the first two considered essential, and the final two ideal:

1. We want to use as few different shapes as possible, ideally just one.

2. The volume and dimensions of the shapes should be small fractions of the total dome volume and radius.

3. The shapes should be roughly polyhedral.

4. The building procedure should be described by a simple algorithm.

Very similar requirements are found in many areas of science, for instance in the construction of grids on spheres in climate research, and in football construction.

It is well known that if three positive integers p, q, r satisfy $1/p + 1/q + 1/r > 1$, then the spherical triangle with angles $A = \pi/p$, $B = \pi/q$, $C = \pi/r$ provides a non-overlapping tiling of the sphere. Since the area of each triangle is $S = \pi(1/p + 1/q + 1/r - 1)$, the half-sphere is divided into $2\pi/S$ segments. The smallest possible such triangle has $p = 2$, $q = 3$, $r = 5$. It is a right angled triangle, which splits the half-sphere into 60 tiles. 30 of them are 'left-handed', and the remaining 30 are the mirrored counterparts of these.

Given a tessellation of the sphere, it is conceptually very easy to split a spherical dome of given thickness. We draw aligned spherical triangles (polygons in general) on the inner and outer sphere, and connect them by straight line segments (see Figure 2).

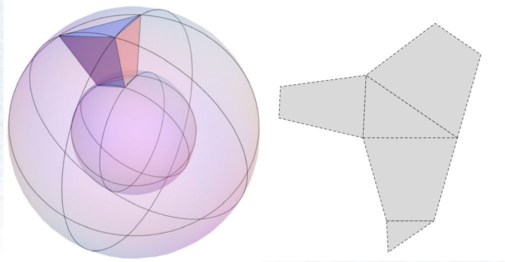

Figure 2 Conversion from spherical triangles into polyhedral dome elements (left) and the net of the dome elements (right)

The Construction Procedure

Combining the above gives us a method for constructing an igloo. To start the dome, we begin with two concentric circles (see Figure 3a). To initialise construction, we first place 12 segments in a non-trivial order (Figure 3b). Note that three elements of the same orientation are placed next to each other, on three different triangle sides. The first row has point reflection symmetry with respect to the centre of the circles. Further blocks are simply reflections of those already placed (Figures 2c and 2d). The most difficult operation is the placement of the final four elements (Figures 2e and 2f), which should ideally all be placed at once.

Paper, gypsum, wet snow and ice bricks have been used to test this procedure on small scales. The igloo has some tendency to come apart under its own weight, so a band around the base must be used.

Conclusion

The '2, 3, 5' spherical triangle above provides a working solution to the igloo building problem, requiring only two different brick forms (the two orientations). Another interesting solution is based on geodesic domes (two different equilateral triangles, 90 bricks). It is still not known whether any single small block type is sufficient to tile the hemispherical dome. Possible search areas are exceptional spherical tilings, and nearly spherical polyhedrons similar to the deltoidal icositetrahedron.

References

[1] Douglas Wilkinson; 1949; *Arctic notebook no. 1: How to build an igloo*; http://www.nfb.ca/film/how_to_build_an_igloo.
[2] Robert J. Mac, G. Dawson; 2003; *Tilings of the Sphere with Isosceles Triangles*; Disc. and Comp. Geom. 30, 467-487; http://cs.stmarys.ca/~dawson/images4.html.

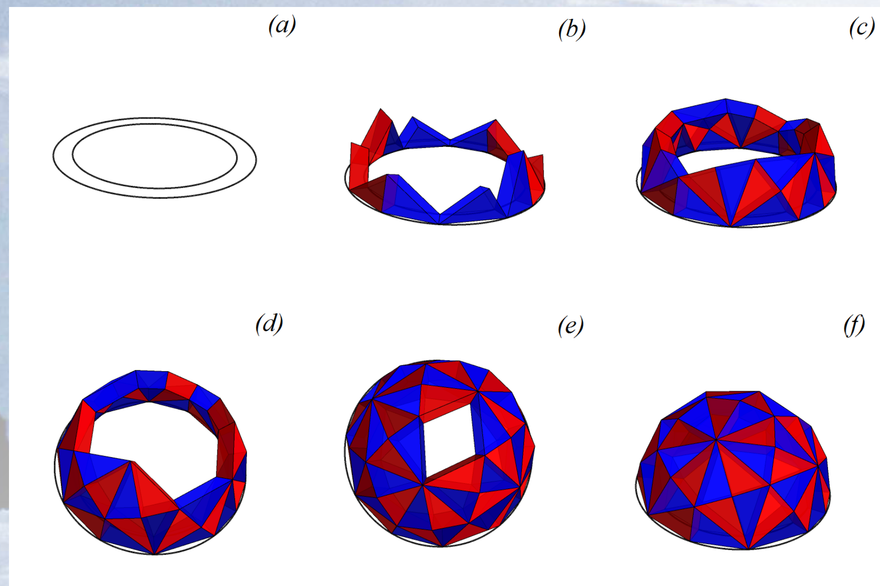

Figure 3 The construction of the igloo, left-handed and right-handed blocks coloured red and blue respectively